On the Relation between Period and Density of Algol-Variables.

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(*Communicated by Prof. H. H. Turner, D.Sc., F.R.S.*)

1. As is well known, a maximum value of the mean density of an Algol-system can be derived from the period (P) and the total duration of eclipse ($2t_0$).* If the orbit is supposed to be circular, this maximum-value D is given by,

$$D = \frac{K}{P^2 \sin^3 nt_0}; \quad n = \frac{2\pi}{P}, \quad . \quad . \quad . \quad (1)$$

where P and t_0 may be expressed in hours, D in the mean density of the Sun as unity.

This value is identical with the *real* mean density (δ) if the two stars are of the same size, and if the inclination of the line of sight to the orbit is zero.

In order to determine the constant K we put—

P = one year = $365{\cdot}25 \times 24^h$; $nt_0 = 32'\ 3''{\cdot}64$ = mean apparent diameter of the Sun; consequently $D = \frac{1}{2}$ and $K = 31{\cdot}17$.

Thus

$$D = \frac{31{\cdot}17}{P^2 \sin^3 nt_0} \quad . \quad . \quad . \quad (2)$$

and $31{\cdot}17$ being $= 1{\cdot}005\ \pi^3$, we can bring this into the simple form—

$$D = \frac{P}{(2t_0)^3}\left(\frac{nt_0}{\sin nt_0}\right)^3 \quad . \quad . \quad . \quad (3)$$

Mériau has shown that D is not very different from δ, if one star is not considerably larger in size than the other.

2. In the *Mitteilungen der Hamburger Sternwarte*, No. 11, Dr. Graff has deduced from his own observations the elements of the orbits of 10 Algol-variables. A slight extension was given to these by Professor Ristenpart (*Ast. Nach.*, No. 4250), who derived from the elements the mean density of the systems by the formula

$$\delta = \frac{a^3 C}{P^2(1 + \kappa^3)} \quad . \quad . \quad . \quad (4)$$

where a is the radius of the relative orbit, κ the radius of the dark satellite, both expressed in the radius of the bright star as unity. If P is given in hours, then $C = \frac{1}{4}K$.

Arranging the stars according to decreasing periods, Ristenpart finds a nearly progressive increase of density; and he adds that this might be expected with regard to formula (4): "Natürlich

* M. Mériau, "Densité des étoiles variables du type d'Algol," *Comptes Rendus de l'Acad. d. Sciences*, vol. 122 (1896), p. 1254.

laufen die Körper um so rascher um einander um, je dichter sie sind."

This argument is, of course, understood, "cæteris paribus"; strictly we can only infer from the results obtained that, with increasing P, the factor $\frac{a^3}{1+\kappa^3}$ in general increases less rapidly than P^2.

3. In order to examine more closely the relation between density and period, I have put together the following list of 38 Algol-stars, for which I could find the duration $2t_0$ of the eclipse, determined exactly enough for the purpose. The fourth column contains the maximum-density D; the next one gives the mean densities δ, derived by Ristenpart from Graff's elements. The slight differences $D-\delta$ are an experimental proof that D is a fairly approximate value of δ. In the sixth column are given the values of $\frac{1}{D}=R$ (="rarity").

Star.	P. h	$2t_0$. h	D.	δ.	R.
RZ Draconis (Z_2)	13·2	2·7	0·848		1·2
ZZ Cygni (Z_6)	15·1	3·5	·358		2·8
U Ophiuchi	20·1	5·1	·211		4·8
RX Herculis (X_2)	21·4	5	·226		4·4
RU Monocerotis (U_2)	21·6	5	·228		4·4
R Canis Mai.	27·3	5	·259		3·9
RZ Cassiopeiæ (Z_2)	28·7	5·5	·203		4·9
Z Draconis	32·6	4·7	·347	·345	2·9
RX Draconis (X_2)	45·5	5	·389		2·6
RW Monocerotis (W_2)	45·8	6·5	·190		5·3
RV Persei (V_2)	47·4	6·5	·194		5·2
δ Libræ	55·8	12	·041		24·4
RS Sagittarii (S_2)	58·0	10·4	·061		16·4
U Cephei	59·8	10	·069		14·5
RY Aurigæ (Y_2)	65·4	8	·138		7·3
143·1907 Andromedæ	66·4	8	·140		7·1
RW Tauri (W_2)	66·4	7·9	·148	·136	6·8
RW Geminorum (W_2)	68·8	12	·046		23·9
β Persei	68·8	10	·077		13·0
Y Cygni	71·9	8	·150		6·7
Z Persei	73·3	11·1	·060	·059	16·7
Y Camelopardalis	79·3	12	·051		19·7
WW Cygni (Z_5)	79·6	11·8	·053	·052	18·9
U Sagittæ	81·1	13·1	·041	·041	24·4
UW Cygni (Z_4)	82·8	10·5	·079	·075	12·7

Star.	P.	$2t_0$.	D.	δ.	R.
	h	h			
U Coronæ	82·8	9·7	·097		10·4
λ Tauri	94·9	10	·100		10·0
Z Herculis	95·8	9·2	·129		7·8
R Aræ	106·2	10·3	·103		9·8
SW Cygni (V_3)	109·5	11·8	·070	·064	14·3
RR Delphini (R_2)	110·4	14	·043		23·3
W Delphini	115·3	17·2	·025	·025	40·0
S Velorum	142·4	15·2	·043		23·3
SY Cygni (X_3)	144·1	19·0	·023	·023	43·5
RY Persei (Y_2)	164·7	26	·010		100·0
VW Cygni (V_5)	202·3	20	·027	·020	37·1
S Cancri	227·6	21·5	·025		40·0
RR Puppis (R_2)	250·3	17	·033		30·4

4. A rough preliminary calculation showed that there can be no question of proportionality between P^2 and R. On the other hand, the product P D oscillated between relatively narrow limits. This seemed to make an investigation of the correlation between P and R more promising.

By taking direct square and product deviations from mean values we find:—*

Including all the 38 stars—

Mean period $\bar{P} = 82{\cdot}66$ hours	Mean "rarity" $\bar{R} = 16{\cdot}95$
Variability in period $\sigma_P = 55{\cdot}60$ hours	Variability in rarity $\sigma_R = 16{\cdot}28$
Coefficient of variability $V_P = {\cdot}672$	Coefficient of variability $V_R = {\cdot}960$

Correlation:

$$r_{R,P} = {\cdot}733 \pm {\cdot}051 \; (p.e.).$$

Probable value of rarity:

$$R = {\cdot}215 \; P - {\cdot}83 \pm 7{\cdot}46 \quad . \quad . \quad . \quad (5)$$

Excluding 4 stars with period greater than 6 days, we have for 34 stars—

Mean period $\bar{P} = 61{\cdot}65$ hours	Mean "rarity" $\bar{R} = 12{\cdot}86$
Variability in period $\sigma_P = 34{\cdot}11$ hours	Variability in rarity $\sigma_R = 10{\cdot}13$
Coefficient of variability $V_P = {\cdot}554$	Coefficient of variability $V_R = {\cdot}788$

Correlation:

$$r_{R,P} = {\cdot}731 \pm {\cdot}054$$

Probable value of rarity:

$$R = {\cdot}217 \; P - {\cdot}51 \pm 4.66.$$

* For the notations used see *M.N.*, Dec. 1908, vol. lxix. pp. 128 ff.

5. Thus the magnitude of correlation seems to indicate a real relationship between period and density, or, more exactly, *between period and duration of eclipse* (see *Note* at end). For a longer period (say > 3 days) we can then put with sufficient accuracy:

$$\mathrm{PD} = \frac{1}{0\cdot216} = 4\cdot63,$$

or, according to (2):

$$\mathrm{P}\sin^3 nt_0 = 6\cdot73.$$

When P is very large, nt_0 must be small, hence $nt_0 = \sin nt_0$, and

$$(2t_0)^3 = \cdot216\ \mathrm{P}^2. \quad . \quad . \quad . \quad . \qquad (6)$$

Applying this, for instance, to the Algol-star 142, 1907 Cassiopeiæ, with a period of $36^{\mathrm{d}}\cdot56 = 877\cdot4$ hours, we find:

$$2t_0 = 55 \text{ hours}; \ \mathrm{D} = 0\cdot005.$$

And, in fact, according to Enebo (*Ast. Nach.*, No. 4241), the duration of eclipse lies between 49·7 and 64·8 hours ($0\cdot003 < \mathrm{D} < 0\cdot007$).

It is hoped that this result, in connection with the relations found by Professor Karl Pearson and Miss Julia Bell, may throw some new light on the nature and evolution of this most interesting class of variable stars.

Specola Vaticana:
1909 *January* 28.

Note.—As was remarked in No. 5, the relation found between P and D is, properly speaking, a relation between the *period* and the observed *duration* of eclipse. This point was especially emphasized by Professor Karl Pearson, to whom the MS. was submitted by Professor Turner. He makes the following interesting remark:—

"With reference to the formula

$$\mathrm{D} = \frac{31\cdot17}{\mathrm{P}^2 \sin^3(nt_0)}; \ n = \frac{2\pi}{\mathrm{P}},$$

if t_0 be a given function of P, it follows that D would be absolutely determined by P. The fundamental question, therefore, is what is the relation between period and eclipse time? I think these must be very closely related. I have accordingly worked them out, and find

$$r_{\mathrm{P},t_0} = +\cdot877.$$

"It will thus, I think, be seen that D being a mathematical function of P and t_0, not the true density, the correlation of D and P is a secondary result of the fundamental relation of the observed quantities P and t_0. In fact, the value of $r_{\mathrm{R,P}}$ can be deduced at once from the values of $\bar{\mathrm{P}}$, $\bar{t}_0$, σ_{P}, σ_{t_0} and r_{P,t_0}."

According to Professor Pearson's remark, putting together the elements of correlation between P and t_0, we get—

Mean period	$\bar{P} = 82^h{\cdot}66$	Mean Eclipse time	$2\bar{t}_0 = 10^h{\cdot}532$
Variability in period	$\sigma_P = 55{\cdot}60$	Variab. in m.E.t.	$\sigma_{2t_0} = 5{\cdot}236$
Coefficient of variab.	$V_P = {\cdot}672$	Coeff. of variab.	$V_{2t_0} = {\cdot}497$

Correlation :

$$r_{P,2t_0} = {\cdot}877 \pm {\cdot}025 \; (p.e.).$$

Probable value of eclipse time,

$$2t_0 = {\cdot}0826 \; P + 3{\cdot}70 \pm 1{\cdot}70 \; (p.e.) \quad . \quad . \quad (7)$$

This value of $2t_0$, between the limits considered, will, of course, not differ substantially from that given by the formula (5) :

$$R = \frac{P^2 \sin^3 nt_0}{31{\cdot}17} = 0{\cdot}215P - 0{\cdot}83. \quad . \quad . \quad (5)$$

This is easily shown by means of the approximate formula (6′) :

$$2t_0 = 0{\cdot}594 \; P^{2/3}. \quad . \quad . \quad . \quad (6')$$

In fact, if we wish to replace the curve

$$y = cx^{2/3} ; \; \alpha < x < \beta$$

approximately by a straight line

$$y = a + bx ; \; \alpha < x < \beta,$$

the best fitting values of a and b, according to the method of least squares, are given by the equations

$$a\int dx + b\int xdx = c\int x^{2/3}dx$$
$$a\int xdx + b\int x^2dx = c\int x^{5/3}dx$$

where the integrals are to be taken between the limits α and β.

Putting $y = 2t_0$, $x = P$, $c = 0{\cdot}594$, $\alpha = 25$, $\beta = 250$, we find

$$2t_0 = 0{\cdot}0797 \; P + 4{\cdot}50$$

in close agreement with (7).

The following table gives in the 2nd. and 3rd. columns the values of $2t_0$, derived from formulæ (5) and (7) :

P	$(2t_0)_5$.	$(2t_0)_7$.	Diff.
h	h	h	h
25	5·2	5·8	− ·6
50	8·5	7·8	+ ·7
75	11·0	9·9	+ 1·1
100	13·4	12·0	+ 1·4
150	17·4	16·1	+ 1·3
200	20·8	20·2	+ ·6
250	24·4	24·3	+ ·1

Radial Movement in Sun-spots. By J. Evershed.

Displacements of the absorption-lines of hydrogen and calcium are frequently observed in all spot disturbances where there is active change in progress, and during the genesis of a spot. These line-shifts may amount to several Ångstrom units in either direction in the spectrum, indicating velocities towards or away from the observer of one or more hundred kilometres per second. Only the elements of the higher chromosphere seem to be affected as a rule, and the movements are seldom maintained for more than a few minutes at a time; also they are rarely found within the penumbral area of a spot.

Recently line-shifts of quite another character have been photographed here. These are permanent, affect most, if not all, of the Fraunhofer lines, and are greatest in the penumbræ.

In some spectra obtained by me this year, in the fourth and fifth orders of a Rowland plane grating of 3·2 inches ruled surface and 14,428 lines to the inch, the lines crossing the spot-band were found to be slightly bent, or inclined one or two degrees as compared with the lines in the neighbouring photosphere. The first plate showing this feature clearly was exposed on January 7, 1909, in the fourth order of the grating. It covers the region $\lambda 4650$ to $\lambda 4790$, and the scale is $1 \text{Å} = 1{\cdot}08$ mm. The total displacement of the lines measured on each side of the penumbra amounts to $0{\cdot}027 \text{Å}$, indicating a receding velocity on the north-west side of the spot of 0·85 km. per second, and an approaching velocity of the same amount on the south-east side. The spot was in latitude 9° north and longitude 31° west of the central meridian, and the position-angle of the north end of the spectrograph slit was 314°.

The appearance of the lines on this plate at once suggested a rotation of the absorbing gases in the penumbra—not a vortex motion, but a rotation of the spot as a whole about a point at its centre. The lines are quite straight over the spot, although inclined to the undisturbed lines; and in the centre of the umbra there is no displacement. The hypothesis of circular motion of any kind has proved, however, to be certainly untenable. From an examination of about 150 spectra, obtained since January 7, and representing seven different spots in the northern hemisphere and four in the southern, the following definite results have been obtained:—

1. All the spots examined show line-shifts of about the same order of magnitude when at the same distance from the centre of the Sun's disc.